PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Aerodynamics of a Spinning Shell. Part II

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VII. *The Aerodynamics of a Spinning Shell.—Part II.*

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§ 1. *Introduction.*

In a previous paper* the authors, with others, have described observations of the angular oscillations of the axis of a 3-inch shell over the first 600 feet from the muzzle of the gun, and from an analysis of the observations have obtained information about the forces due to the air. In the experiments, shells were fired from two guns giving different degrees of axial spin to the shell. While the shells fired from the gun giving the more rapid spin were all stable, most of the shells from the other gun were slightly unstable, this condition being shown by the much larger maximum yaw† developed. These unstable rounds were not analysed in (A) as no suitable method of doing so had then been devised.‡ The analysis of these rounds, about one-third of the number fired, forms the subject of the present paper; the results confirm those of (A) and provide some additional information.

The information as to the force system obtained from the stable rounds was confined to yaws up to 7 degrees or perhaps 10 degrees; by analysis of the unstable rounds this information is extended, though in a fragmentary manner, over the region up to 35 degrees of yaw. On the other hand, no information has been derived from the observed damping of the unstable rounds. The observations are, in respect of the damping, clearly in qualitative agreement with the theory and results of (A), but no method has been devised of making a quantitative analysis of the damping.

The force system on a model shell was also determined at low velocity in the wind channels of the National Physical Laboratory. The results are shown in fig. 2 of (A)

* "The Aerodynamics of a Spinning Shell," 'Phil. Trans.,' A, vol. 221, p. 295 (1920). This paper will be cited as (A). The experiments analysed here and in (A) were carried out for the Ordnance Committee, and the results are published with their sanction.

† The "yaw" is the angle between the axis of the shell and the direction of motion of its centre of gravity.

‡ As will appear later, the ordinary solution in elliptic functions of the equations of motion of a top is not adequate for this purpose in the case of large yaws.

and have been used in both papers for extending the results down to low velocities, as in figs. 1 and 2, here.

Shells of four different types, I.–IV. were used. Types I.–III. were of the same external shape (form A), with three different positions of the centre of gravity. Type IV. was of a different external shape (form B). The details are given in (A).*

The experimental data, which have already been discussed in (A), consist of the mass, principal moments of inertia, and position of the centre of gravity for each type of shell ; rough values of the forward velocity over the whole range of 600 feet from the muzzle of the gun ; the spin of the shell, deduced from the rifling of the bore ; the yaw and orientation of the shell's axis at a number of points along the range, deduced from the shape and orientation of the holes punched by the shell in cardboard targets. These cards were set up at intervals of about 60 feet for all the unstable rounds, and it appears from figs. 12 of (A) and figs. 3 and 4 of this paper that they were close enough together for satisfactory curves to be drawn through the observed points representing the variation of the yaw δ and its azimuth ϕ over the whole range.

§ 2. *The Equations of Motion.*

It is convenient to recapitulate the notation of (A). Suppose that OA denotes the direction of the axis, OP the direction of motion of the centre of gravity of the shell ; then AOP $= \delta$, and ϕ is the angle that the plane AOP makes with a fixed plane through OP. M $(= \mu \sin \delta)$ is the couple in the plane AOP which tends to increase δ, A, B and N are the principal moments of inertia and the axial spin of the shell, and we write $\Omega = \mathrm{AN/B}$. Then the equations of motion will be taken in the form†

$$\delta'^2 + \phi'^2 \sin^2 \delta + \int \frac{2\mu}{\mathrm{B}}\, d \cos \delta = \mathrm{E}, \qquad (1)$$

$$\phi' \sin^2 \delta + \Omega \cos \delta = \mathrm{F}, \qquad (2)$$

where E and F are constants. The various assumptions underlying equations (1) and (2) are discussed in detail in (A). If μ is constant the equations are, of course, of the same form as the ordinary integrals of energy and angular momentum for a spinning top, and the complete solution in elliptic functions is standard.

When M is an arbitrary (odd) function of δ the top solution no longer applies, but a solution in elliptic functions is still possible if M has the form $\mathrm{X} \sin \delta \{1 - \mathrm{Y}(1 - \cos \delta)\}$, where X and Y are constants. This more general form allows the first two terms in the expansion of an arbitrary M to be catered for and can represent M adequately

* *Loc. cit.*, p. 316 and fig. 6. See also fig. 1, here.

† (A), *loc. cit.*, p. 334, equations 3.404, 3.405. For the underlying assumptions see (A) Part I., pp. 301 *sqq.*, 311 *sqq.* These equations are, strictly speaking, not referred to fixed axes, but are approximate equations referred to axes changing direction with OP. Dashes denote differentiations with respect to the time t.

over a wider range of values of δ. By suitably adjusting X and Y, which define the couple, and the initial conditions, a curve showing the variation of δ with the time can be obtained which agrees closely with observations over a complete half-period, so that the above expression for M appears to be adequate up to values of δ of 35 degrees.* Our original approximation with $Y = 0$ fails in general when $\delta > 10$ degrees. The observed curves suffice to determine X and Y for each round, and as observations were taken for a number of different values of the muzzle velocity, M is determined by the experiments over a limited range, as a function of the two variables, v, the velocity of the shell, and δ.

In solving the equations of motion it is convenient to express the couple by means of the non-dimensional coefficients s and q defined by the equation

$$M = \frac{B\Omega^2 \sin \delta}{4s} \{1 - 4qs(1 - \cos \delta)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It will appear that the motion with δ permanently zero is stable or unstable according as $s > 1$ or $s < 1$. For the rounds here analysed, s lies between $1 \cdot 06$ and $0 \cdot 83$.

In expressing the results in a standard form it is convenient to use a different non-dimensional coefficient f_M, which is independent of the mass, moments of inertia, size and velocity of the shell, and depends only on the shape of the shell and the non-dimensional variables v/a and δ, where a is the velocity of sound. This is defined by the equation†

$$M = \rho v^2 r^3 \sin \delta \, f_M (v/a, \delta), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ρ is the air-density, r the radius of the shell, and the quantities involved are expressed in consistent units.

According to (3), f_M is practically constant so long as $\delta < 7$ degrees, and the value of f_M $(v/a, 0)$ is strictly comparable with similar values obtained in (A) by analysis of the stable rounds on the assumption that f_M is independent of δ.

§ 3. *Final Results of the Experiment.*

Fig. 1 shows curves of $f_M (v/a, 0)$ as a function of v/a for the four types of shell corrected for the effect of the cards‡. They are reproduced without alteration from figs. 4 and 5 of (A) and represent the results for the stable rounds.§ The values derived from the present analysis of the unstable rounds are plotted for comparison;

* When the yaw exceeds 30 degrees the fit is less satisfactory (*e.g.*, III., 11–13).

† *Loc. cit.*, p. 302, equation 1.103.

‡ § 10 below.

§ The curve for type II. is not actually given, but the data for drawing it can be found in (A) (fig. 13, p. 352).

they show remarkably good agreement between the new results and the old. This confirms the substantial truth of the whole theory; in particular, the agreement of the results for the two twists of rifling verifies that the couple M is unaffected by a change in axial spin in the ratio 3 to 4.

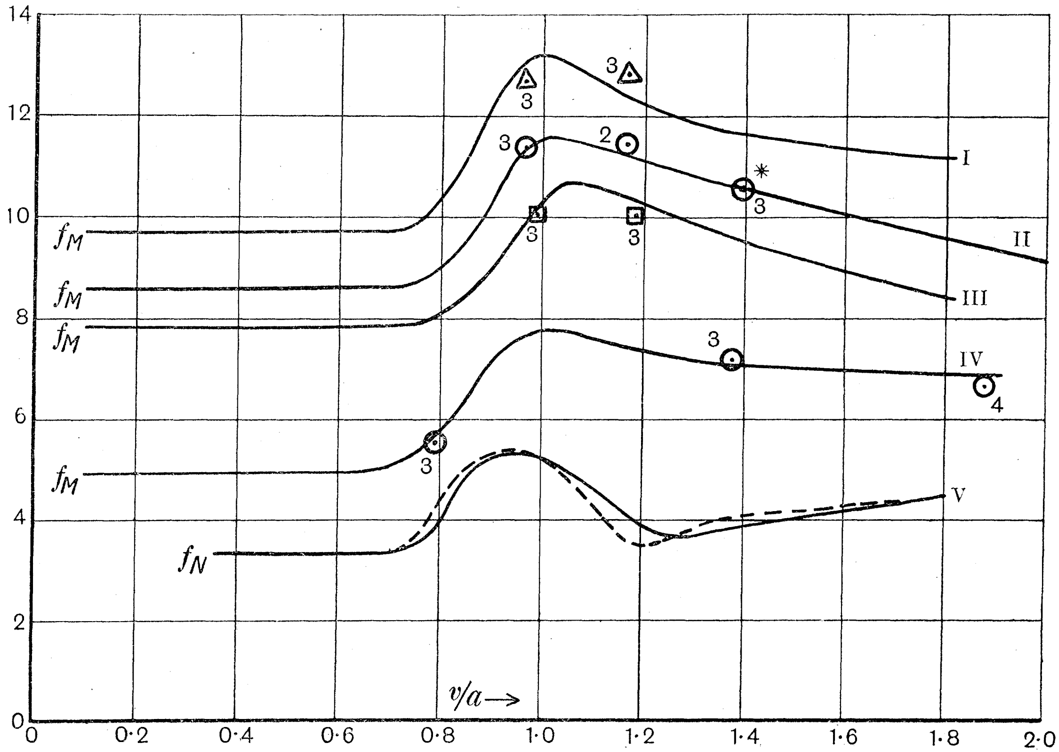

Fig. 1. Values of the couple coefficient $f_M(v/a, 0)$ and the normal force coefficient $f_N(v/a, 0)$ plotted against v/a, corrected for the effect of the cards. The full curves for f_M for types I.–IV. are reproduced without alteration from (A). The plotted points show the means of observations here analysed for the first and second half-periods, and the numbers against them show the number of rounds contributing to each mean. The group marked * was analysed in (A) by a different method with identical results. The origin of f_M for type IV. is displaced downwards 4 units.

The full curve (V) for f_N (shells of form A) represents the complete results of the experiment. The dotted curve reproduces the partial results of (A).

Curve I.—f_M for 3-inch shells of form A with centre of gravity 4·20 inches from the base. (Type III.)
Curve II.—The same with centre of gravity 4·73 inches from the base. (Type I.)
Curve III.—The same with centre of gravity 5·08 inches from the base. (Type II.)
Curve IV.—The same for form B with centre of gravity 4·965 inches from the base. (Type IV.)

Curves of $f_M \sin\delta$, the complete moment coefficient, considered as a function of both variables v/a and δ corrected for the effect of the cards, are plotted against δ in fig. 2. The information is somewhat fragmentary: in addition to wind channel results, values of f_M, when $\delta > 10$ degrees, are available for shells of type I.–III. for two velocities near $v = a$. For type IV. (pointed shells) values of f_M for large δ are wanting in this region, but exist for two high velocities, and one less than a.

Some general information can be deduced from these curves. At high velocities ($v/a = 2\cdot0$) and large values of δ the couple coefficient actually falls below its low

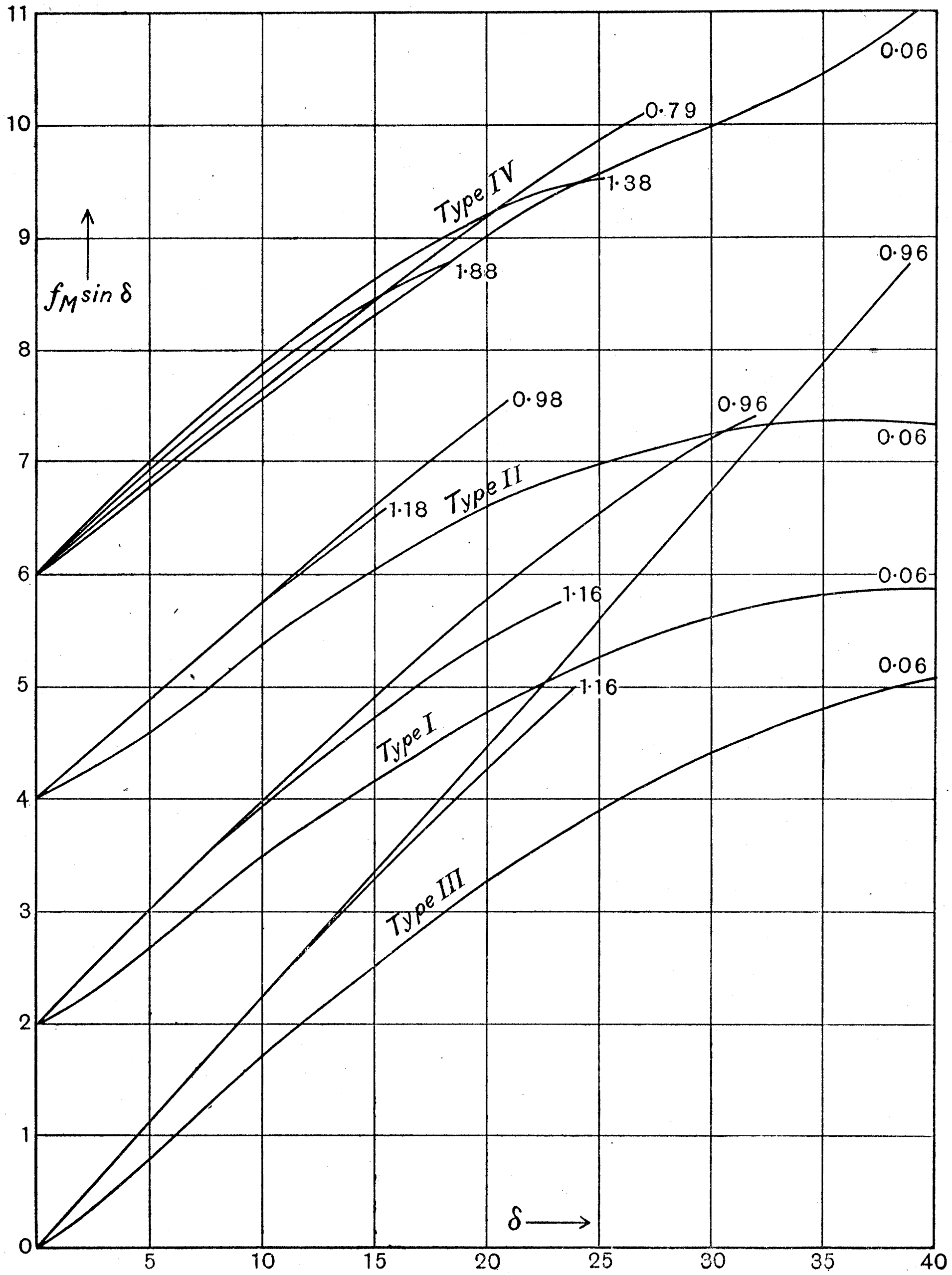

Fig. 2. Curves of the complete couple coefficient $f_M(v/a, \delta) \sin \delta$ against δ for various values of v/a (as shown against the curves). The curves stop at the greatest value of δ for which observations are available.

The origins of all the curves except for type III. are displaced upwards and can be recovered from the fact that all curves pass through their respective origins. The scale of δ is in degrees.

velocity (wind channel) value (fig. 2); the curve of the couple coefficient against yaw has here a large curvature downwards. It is, on the other hand, almost straight in the region of the velocity of sound.

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We may notice also the peculiar behaviour of rounds III. (11–13), fig. 3,* in which the motion seems to become quite irregular from round to round after the first maximum yaw is attained. As these are the only rounds in which a yaw of 35 degrees or more is developed, it seems likely that some peculiar change in the type of the airflow occurs at or about this angle, analogous to the phenomenon of "burbling."†

Rounds IV. (10–12) have the smallest velocity of any group fired; the agreement of f_M for these rounds with wind channel results for the whole range of δ, figs. 1 and 2, is very satisfactory.

The new values of f_M $(v/a, 0)$ can be used to correct slightly the mean curves of f_M, and from the modified curves the value of f_N, the coefficient of the force normal to the shell's axis at small yaw, may be re-determined by the method used in (A), § 1·13. The re-determined curve for f_N is shown in fig. 1. The values differ only slightly from the former values, and the main features of the f_N curve are fully confirmed.

The final results of the complete experiment, stable and unstable rounds alike, corrected for the effect of the cards, have been combined together to give the final values of f_M $(v/a, 0)$ for shells of types I. and IV. and f_N $(v/a, 0)$ for type I. shown in Table I., which replaces the corresponding Table I. of (A).

Table I.—Final values of f_M $(v/a, 0)$ and f_N $(v/a, 0)$ for shells of type I. and f_M $(v/a, 0)$ for shells of type IV., embodying the results of the whole experiment. The effect of the cards has been corrected for as far as possible, and this table supersedes Table I. of (A).

v/a.	Shells of type I.		Type IV.
	f_M $(v/a, 0)$.	f_N $(v/a, 0)$.	f_M $(v/a, 0)$.
Wind channel	8·57	3·34	8·95
0·7	8·6	3·35	9·0
0·8	9·05	4·0	9·7
0·9	10·35	5·2	11·1
1·0	11·55	5·25	11·75
1·1	11·55	4·7	11·6
1·2	11·25	3·9	11·4
1·3	10·9	3·7	11·25
1·4	10·55	3·85	11·1
1·5	10·3	4·0	11·0
1·6	10·05	4·15	10·95
1·7	9·85	4·3	10·9
1·8	9·65	4·5	10·8
1·9	9·4	—	10·75
2·0	9·15	—	—

* Also (A), p. 349, fig. 12B.

† This term is commonly applied to the sudden increase of turbulence behind an aerofoil at the critical angle.

§ 4. *The Solution of the Equations of Motion.*

We shall now solve the equations of motion (1) and (2), assuming that M is of form (3). We take first the case of rosette motion, in which zero values of δ can occur, so that we may assume the initial conditions

$$\delta = 0, \qquad \delta' = b\Omega.$$

Eliminating ϕ' and writing $\sin \frac{1}{2}\delta = y$ we get

$$4y'^2 = \Omega^2 \{b^2(1-y^2) - y^2 + y^2(1-y^2)(1/s - 4qy^2)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The right-hand side is a cubic in y^2 whose roots are such that it may be written in the form

$$4y'^2 = 4q\Omega^2(h^2+y^2)(a^2-y^2)(f^2-y^2), \qquad (1/f^2 < 1 < 1/a^2). \quad . \quad . \quad . \quad (6)$$

Formulæ connecting* h, a and f with b, q and s may be obtained most conveniently by putting $y^2 = 1$, $y^2 = 0$, and by differentiating with respect to y^2 and putting $y^2 = 0$. The resulting formulæ are

$$4q(1+h^2)(1-a^2)(f^2-1) = 1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$4qa^2h^2f^2 = b^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$4q(a^2f^2 - a^2h^2 - h^2f^2) = -b^2 + 1/s - 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

A solution of (6) is obtained by assuming, in the usual notation of Jacobian elliptic functions,

$$y^2 = \frac{a^2 \operatorname{cn}^2 u}{1 - g^2 \operatorname{sn}^2 u}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where the constant g and the modulus k of the elliptic functions remain to be determined. If we solve (10) for $\operatorname{cn}^2 u$ and differentiate, we get

$$-\operatorname{sn} u \operatorname{cn} u \operatorname{dn} u \, u' = \frac{a^2(1-g^2)yy'}{(a^2-g^2y^2)^2},$$

leading to

$$y'^2 = \frac{u'^2}{a^4(1-g^2)}(a^2-g^2y^2)(a^2-y^2)\{a^2(1-k^2)+(k^2-g^2)y^2\}.$$

Comparing this with (6) we may write $u' = \pm\lambda\Omega$, λ constant, and obtain

$$f^2 = a^2/g^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$h^2 = \frac{a^2(1-k^2)}{k^2-g^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$q = \frac{\lambda^2 g^2(k^2-g^2)}{a^4(1-g^2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

* This a has, of course, no connection with the velocity of sound.

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Since $y = 0$ when $t = 0$, we may write $u = \mathrm{K} - \lambda\Omega t$, where K is the complete elliptic integral of the first kind to modulus k. Treating Ωt as independent variable, the final form of our solution contains four constants a, λ, k and g, which must be completely determinable in terms of s, q and b, so that there must be an independent relation between a, λ, k and g by which any one can be found when the other three are known. To determine them it would be necessary to solve (5), the original cubic in y^2. To analyse the experiments, however, we have to solve the inverse problem of determining s, q and b when a, λ, k and g are known. In practice g is small so that, as a first approximation, we may use the following simplified form of (10):—

$$y = a \operatorname{cn} (\mathrm{K} - \lambda\Omega t), \quad \ldots \ldots \ldots \quad (14)$$

where a, k and λ may be treated as independent. By fitting a curve of this type to the curve of observed values of y ($\sin \frac{1}{2}\delta$) against Ωt, we can determine the constants a, λ and k. It is at once clear that $a = \sin \frac{1}{2}\alpha$, where α is the maximum yaw, but we shall continue to call this constant a for shortness. The value of g can then be obtained from the identical relation in terms of a, k, λ, and the curve re-calculated by formula (10) if g is large enough to make it worth while to do so. After that the values of a, λ and k could be re-adjusted and the process repeated. Theoretically, we could presumably arrive at the precise solution in this manner by a limiting process. Practically, in nearly every case, the first approximation with $g = 0$ is all that is required.

The values of s, q and b are given by simple formulæ in terms of a, λ, k and g. From (8) and (9) we get

$$1/s - 1 = 4q\,(a^2h^2f^2 + a^2f^2 - a^2h^2 - h^2f^2);$$

on substituting for q, h^2 and f^2 from (11)–(13), this reduces to

$$1/s - 1 = 4\lambda^2 \left\{ -\cos 2\kappa + \frac{\cos^2 \kappa\,(a^2 - 3g^2)}{1 - g^2} \right\}, \quad \ldots \ldots \quad (15)$$

where $k = \sin \kappa$.* In practice either a or $\cos \kappa$, or both, are small and g is of the same order as a; the second term inside the bracket may then be neglected in determining s, in which case the value of g is not required. This, as we shall see, is really a consequence of the smallness of b, its mean value in practice being about 0·015. For q write equation (7) in the form

$$4q + \frac{1}{1 - a^2} = 4q\,(h^2f^2 - h^2 + f^2),$$

and substitute for q, h^2 and f^2 in the right-hand side, getting

$$4q = -\frac{1}{1 - a^2} + \frac{4\lambda^2}{a^2} \left\{ \sin^2 \kappa + \frac{\cos^2 \kappa\,(a^2 - 2g^2)}{1 - g^2} \right\}. \quad \ldots \ldots \quad (16)$$

* This κ will not be confused with the κ of (A), *loc. cit.*, p. 328, which is the damping coefficient depending on the cross-wind force.

Finally equation (8) gives

$$b^2 = \frac{4\lambda^2 a^2 \cos^2 \kappa}{1-g^2}. \quad \quad (17)$$

The equation for g is obtained by substituting for q, h^2 and f^2 in (7), which becomes

$$4\lambda^2 (1-a^2)(a^2-g^2)\{a^2-g^2+(1-a^2)k^2\}-a^4(1-g^2) = 0; \quad \quad (18)$$

this is a quadratic for g^2, whose solution may be written

$$g^2 = \sin^2 \tfrac{1}{2}\alpha \left(1-\frac{\tan\frac{1}{2}\theta}{2\lambda}\right), \qquad (\sin\tfrac{1}{2}\alpha = a), \quad \quad (19)$$

where θ is given by

$$\cot\theta = \lambda k^2 \cot^2 \tfrac{1}{2}\alpha - (\tan^2 \tfrac{1}{2}\alpha)/4\lambda. \quad \quad (20)$$

The ambiguity is settled in practice by the fact that g^2 must be small if equation (14) is to be taken as a first approximation to the solution. In practice, as we have said, b is small. Valuable information as to the nature of the solution is, therefore, obtainable by considering its limit as $b \to 0$. This gives us a guide as to the actual relative order of all terms.

Let us suppose then that $b \to 0$, that s and q are definite constants, and let us assume that g is of the same order as a, which by (19) and (20) must be the case unless $\lambda \to 0$. Then equation (17) shows that $\lambda^2 a^2 \cos^2 \kappa/(1-g^2) \to 0$, and, therefore, in the limit,

$$1/s-1 = -4\lambda^2 \cos 2\kappa. \quad \quad (21)$$

Equation (21) shows that, if $s \neq 1$, $\lambda \to 0$ is impossible. Hence in all cases ($s \neq 1$) our assumption as to g is justified and $a \cos \kappa \to 0$. This also justifies our previous statement concerning (15). There are now two cases according as $s < 1$ or $s > 1$.

Case (i).—$s < 1$. We are supposing that $s-1$ is fixed, so that as $b \to 0$, $s-1$ is large compared to b. To satisfy the signs of (21) we must have $\kappa > 45$ degrees. This implies $\sin^2 \kappa > \frac{1}{2}$, so that (16) becomes in the limit

$$4q = -\frac{1}{1-a^2}+\frac{4\lambda^2 \sin^2 \kappa}{a^2}.$$

It follows that $a \to 0$ is impossible, and therefore $\cos \kappa \to 0$, $\kappa \to 90$ degrees, and a and λ tend to definite non-zero limits. The limiting forms for $1/s$, q and g^2 are easily found to be

$$1/s-1 = 4\lambda^2, \quad \quad (22)$$

$$4q = -\sec^2 \tfrac{1}{2}\alpha + 4\lambda^2 \operatorname{cosec}^2 \tfrac{1}{2}\alpha, \quad \quad (23)$$

$$g^2 = \sin^2 \tfrac{1}{2}\alpha \,\{1-(\tan^2 \tfrac{1}{2}\alpha)/4\lambda^2\}, \quad \quad (24)$$

in which we have replaced a by $\sin \frac{1}{2}\alpha$. These formulæ are good approximations in practice, when s is not too close to unity. The half-period ΩT* ($= K/\lambda$) tends to infinity, but so slowly that no difficulty occurs in practice. Since α, the maximum yaw, does not tend to zero with b, the initial disturbance, we have here what may be called *the unstable case.*

Case (ii).—$s > 1$. We now must have $\kappa < 45$ degrees and, therefore, as $\cos^2 \kappa > \frac{1}{2}$, $a \rightarrow 0$ at the same rate as b; this is *the stable case* in the usual sense. Equation (16) shows further that $\kappa \rightarrow 0$, and, therefore, by (15), λ has a definite limit determined by

$$1 - 1/s = 4\lambda^2.$$

For given q equations (16)–(20) determine the limiting ratios of $b : \sin \frac{1}{2}\alpha : k : g$.

The case in which $s = 1$ and $b \rightarrow 0$ can be treated in a similar way. It is found that $\kappa \rightarrow 45$ degrees and a and λ both tend to zero like $\sqrt{b}$.

§ 5. *Rounds with a Non-Zero Minimum Yaw.*

We shall only consider cases in which the minimum yaw β is small, and shall take as initial conditions

$$\delta = \beta, \qquad \delta' = 0, \qquad \phi' \sin \delta = b_1 \Omega.$$

The equations of motion become

$$\phi' \sin^2 \delta - \Omega b_1 \sin \beta + \Omega (\cos \beta - \cos \delta) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\delta'^2 + \phi'^2 \sin^2 \delta - \Omega^2 b_1^2 + \int_\beta^\delta \frac{2\mu}{B} d \cos \delta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

If we write $y_1^2 = \sin^2 \frac{1}{2}\delta - \sin^2 \frac{1}{2}\beta$, y_1 vanishes initially and the equation for y_1 may be written

$$4y'^2_1 = \Omega^2 \{b_1^2 (\cos \beta - y_1^2) - y_1^2 - b_1 \sin \beta$$
$$+ (\sin^2 \tfrac{1}{2}\beta + y_1^2)(\cos^2 \tfrac{1}{2}\beta - y_1^2)[1/s - 4q(\sin^2 \tfrac{1}{2}\beta + y_1^2)]\}. \quad . \quad . \quad . \quad (27)$$

We identify (27) with the equation

$$4y'^2_1 = 4q\Omega^2 (h_1^2 + y_1^2)(a_1^2 - y_1^2)(f_1^2 - y_1^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

in which $a_1^2 = \sin^2 \frac{1}{2}\alpha - \sin^2 \frac{1}{2}\beta$. Equations (10)–(13) retain their form, and (7)–(9) become

$$(b_1 \sin \tfrac{1}{2}\beta + \cos \tfrac{1}{2}\beta)^2 = 4q(h_1^2 + \cos^2 \tfrac{1}{2}\beta)(f_1^2 - \cos^2 \tfrac{1}{2}\beta) \cos^2 \tfrac{1}{2}\alpha, \quad . \quad . \quad . \quad (29)$$

$$b^2 = (b_1 \cos \tfrac{1}{2}\beta - \sin \tfrac{1}{2}\beta)^2 = 4q(h_1^2 - \sin^2 \tfrac{1}{2}\beta)(f_1^2 + \sin^2 \tfrac{1}{2}\beta) \sin^2 \tfrac{1}{2}\alpha, \quad . \quad . \quad . \quad (30)$$

$$-b_1^2 + (1/s) \cos \beta - 1 - 4q \sin^2 \tfrac{1}{2}\beta (2 \cos^2 \tfrac{1}{2}\beta - \sin^2 \tfrac{1}{2}\beta) = 4q(a_1^2 f_1^2 - h_1^2 a_1^2 - h_1^2 f_1^2). \quad . \quad (31)$$

* T is the time interval between a zero or minimum and an adjacent maximum of the yaw.

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The solution is

$$y_1^2 = \frac{a_1^2 \operatorname{cn}^2 u}{1-g^2 \operatorname{sn}^2 u}, \qquad (u = \mathrm{K}-\lambda\Omega t),$$

or

$$\sin^2 \tfrac{1}{2}\delta - \sin^2 \tfrac{1}{2}\beta = \frac{(\sin^2 \frac{1}{2}\alpha - \sin^2 \frac{1}{2}\beta) \operatorname{cn}^2 u}{1-g^2 \operatorname{sn}^2 u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The values of s and q in terms of λ, $\sin \frac{1}{2}\alpha$, k and g will differ from the values they had before by terms of the order $\sin^2 \frac{1}{2}\beta$ or $b \sin \frac{1}{2}\beta$ which are negligible in practice compared to a_1^2. Hence the previous solution may be applied provided that (32) replaces (10) in calculations of the curve of $\sin \frac{1}{2}\delta$ as a function of Ωt. For convenience in computing, (32) may be put in the form (neglecting g^2)

$$\sin \tfrac{1}{2}\delta = \frac{\sin \frac{1}{2}\alpha \cos \chi}{\sin \theta} = \frac{\sin \frac{1}{2}\beta \sin \chi}{\cos \theta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where

$$\tan \theta = \sin \tfrac{1}{2}\alpha \cot \chi / \sin \tfrac{1}{2}\beta, \qquad \cos \chi = \operatorname{cn} u.$$

Formula (17) remains a valid approximation for b^2 provided b^2 is defined by (30).

§ 6. *A Discussion of the Probable Effects of Damping, and Other Factors Omitted in the Foregoing Solution.*

Up to this point we have assumed that the motion in yaw is exactly periodic with half-period ΩT. This would be exactly true if the couple M were a function of δ only, OP a fixed straight line, and no other couples existed. In actual fact, M is a function of the forward velocity and therefore of the time; OP changes direction under the influence of gravity and the cross-wind force, and other couples besides M act on the shell, depending on the angular velocity of the shell. All these factors cause progressive changes in the curve of yaw from period to period; for the case of the stable rounds they have been discussed at length in (A), where it is shown that they do not appreciably affect the determination of M at any velocity for small values of δ. In particular, the effect of gravity is almost entirely allowed for by using (as we do) the true yaw and not the angle between the axis of the shell and some *fixed* straight line. As explained in (A)* the shape of the hole in the cards determines the true yaw and not the angle between the axis of the shell and the normal to the card.

There is no reason to expect that any of these damping effects will be relatively more important for an unstable than for a stable shell, except for the change of M with the velocity. Although the change in velocity over a single period is always small, yet when s is less than or nearly equal to unity a small change in M will cause a fairly

* *Loc. cit.*, p. 318, footnote.

large change in the type of motion. This is shown clearly in fig. 3, round III. 16, where the decrease in amplitude and the change from an unstable to a stable type is quite marked, but still not sufficient to introduce any error in the determination of the couple for a single half-period. The effect is illustrated by the change in s in successive half-periods (Table III.), which is in general in the direction, and roughly of the amount, required by theory.

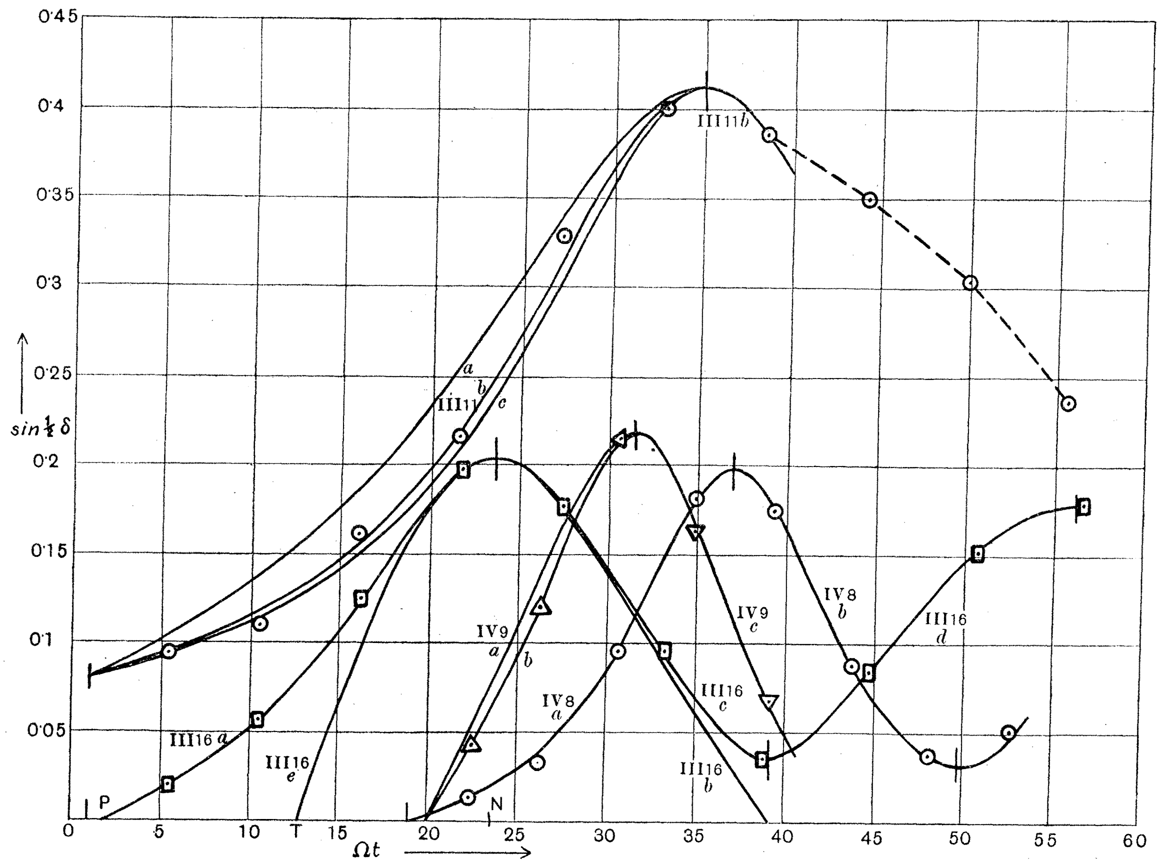

Fig. 3. The observed and calculated motion in yaw, compared for selected rounds. The plotted points show the observed values of sin $\frac{1}{2}\delta$ plotted against Ωt for rounds III. (11, 16) and IV. (8, 9). The continuous curves are the result of calculations described in detail in §§ 6, 8. Short vertical lines mark the positions of maxima and minima, and the origins of co-ordinates. The values of the constants used are as follows :—

III. 11. (*a*) $\kappa = 80$ degrees ; (*b*) $\kappa = 85$ degrees, $g^2 = 0$; (*c*) $\kappa = 85$ degrees, $g^2 = -0{\cdot}185$.
III. 16. (*a*) $\kappa = 80$ degrees ; (*b*) $\kappa = 60$ degrees, $\beta = 0$; (*c*) $\kappa = 60$ degrees, $\sin \frac{1}{2}\beta = 0{\cdot}037$; (*d*) $\kappa = 40$ degrees, $\sin \frac{1}{2}\beta = 0{\cdot}037$; (*e*) $\kappa = 0$, with third order contact with (*a*) at maximum.
IV. 8. (*a*) $\kappa = 85$ degrees ; (*b*) $\kappa = 75$ degrees, $\sin \frac{1}{2}\beta = 0{\cdot}034$.
IV. 9. (*a*) $\kappa = 60$ degrees ; (*b*, *c*) $\kappa = 70$ degrees.

It appears that a change in M with v cannot alter an initial rosette motion into one with non-zero minimum yaw. This alteration, as in the stable case, must be due to the other couples depending on the angular velocity of the axis, and to the sideways

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motion of the centre of gravity, which function as damping forces as explained in (A).* No means of dealing with these effects theoretically has yet been devised for the case of large yaw; the observed changes are clearly of the general type which one's experience of the stable case would lead one to predict.

§ 7. *The Motion in* ϕ.

It is not difficult to write down a formal expression for the motion in ϕ. If we take equation (25) and substitute for $\sin\frac{1}{2}\delta$ from equation (32) we obtain after reduction

$$\phi' = \frac{\Omega}{1+\cos\delta} + \frac{b\Omega\sin\frac{1}{2}\beta}{(1+\cos\delta)\,y^2}, \quad \ldots \ldots \ldots \quad (33)'$$

$$= \frac{\Omega}{1+\cos\delta} + \frac{b\Omega\sin\frac{1}{2}\beta}{1+\cos\delta}\,\frac{1-g^2\,\mathrm{sn}^2 u}{(1-g^2)\sin^2\frac{1}{2}\beta\,\mathrm{sn}^2 u+\sin^2\frac{1}{2}\alpha\,\mathrm{cn}^2 u},$$

where b is defined by (30). Now even when α is as big as 30 degrees there is still only a maximum difference of 7 per cent. between $1+\cos\delta$ and 2, and this maximum is only effective for a short part of each period. Hence, for almost all purposes, we are still justified in replacing $1+\cos\delta$ by 2†. In order to integrate this equation we notice that $\lambda\Omega dt = -du$ and that $t=0$ or $u=\mathrm{K}$ corresponds to the minimum. Thus,

$$\phi = \phi_0 + \tfrac{1}{2}\Omega t + \frac{b\sin\frac{1}{2}\beta}{2\lambda}\int_u^{\mathrm{K}} \frac{(1-g^2\,\mathrm{sn}^2 u)\,du}{(1-g^2)\sin^2\frac{1}{2}\beta\,\mathrm{sn}^2 u+\sin^2\frac{1}{2}\alpha\,\mathrm{cn}^2 u}. \quad \ldots \quad (34)$$

This equation contains an elliptic integral of the third kind which can be evaluated in θ-functions. We have not, however, made this evaluation or calculated any actual ϕ-curves from (34) mainly because it does not appear that any further information as to the forces acting on the shell would be obtained thereby. We shall content ourselves in this paper with a statement of sufficient theoretical results to show that the observed ϕ-curves are qualitatively of the form to be expected from (34). A more detailed discussion of these curves, however, would, we think, be of some interest.

It is convenient to treat the motion by using the variable $\psi = \phi - \phi_0 - \frac{1}{2}\Omega t$. When β is zero, ϕ' will be constant and equal to $\frac{1}{2}\Omega$ to our present approximation; with y and ψ as polar co-ordinates the motion then consists of an oscillation in a straight line through the origin, for $\psi' = 0$. In the general case we may eliminate dt between equations (33)′ and (28), and on substituting for b from (30) obtain an equation for $dy/d\psi$ in the form‡

$$\left(\frac{dy}{d\psi}\right)^2 = y^2\left(\frac{y^2}{\sin^2\frac{1}{2}\beta}-1\right)\left(1-\frac{y^2}{\sin^2\frac{1}{2}\alpha}\right)\left(1+\frac{y^2}{h_1^2-\sin^2\frac{1}{2}\beta}\right)\left(1-\frac{y^2}{f_1^2+\sin^2\frac{1}{2}\beta}\right). \quad . \quad (35)$$

If we assume that h_1^2 and f_1^2 are large compared to $\sin^2\frac{1}{2}\alpha$, the last two brackets

* In particular see p. 313.

† It is easy to estimate the precise effect of this approximation in the simple case of the rosette motion. The error caused is always very small.

‡ In deducing (35) we replace $1+\cos\delta$ by 2.

in (35) are effectively unity, and the equation reduces to that of an ellipse in polar co-ordinates with axes $\sin\frac{1}{2}\alpha$, $\sin\frac{1}{2}\beta$. This tends to a straight line as the limiting form when $\beta \rightarrow 0$.

The same (simplified) relation between δ and ψ was obtained in (A)* for the case of small yaw only. The elliptic motion may be calculated most easily by recognising that ψ is the auxiliary angle θ of equation (33) so that

$$\phi = \phi_0 + \tfrac{1}{2}\Omega t + \theta.$$

The conditions under which this approximation is valid may be seen by reference to equations (11) and (12) which are satisfied by f_1^2, h_1^2 and a_1^2. We notice that f_1^2 is never small compared to unity and tends to infinity as q, and therefore g tends to zero; while h_1^2 may be comparable with a_1^2 unless k is small. Thus the approximation is really only valid for $s > 1$, in which case it applies even if β is not small compared to α. Finally, if $1-k^2$ is small, equation (12) shows that h_1^2 will be small compared to a_1^2, and the third bracket in (35) will be more important than the second near a minimum of y; this indicates that the shape of the (y, ψ) curve there approximates to an hyperbola instead of to an elongated ellipse; the curve may also no longer be re-entrant, the total change of ψ in one period differing from $\pm\pi$.

Examples of all these results may be seen in fig. 4, in which the observations for three different rounds are plotted with (y, ψ) as polar co-ordinates. For round I. (5), for which β is small and the shell just stable, we find the expected elongated ellipse-like curve, with a slowly-developing minor axis caused by the damping factors. In III. (16) $\kappa = 80$ degrees for the first half-period, diminishing to 40 degrees for the third; a considerable minimum yaw developes and the curve is less like an ellipse, though the maxima are still nearly 180 degrees apart. Finally, in IV. (8) $\kappa = 85$ degrees, falling to 75 degrees, and the shape of the curve near the minimum clearly resembles an hyperbola; we may guess then the angle between the two maxima is somewhere about 100 degrees instead of 180 degrees.

Lastly, a word must be said about the observational determination of Ω, which is, of course, theoretically determined by the muzzle velocity, twist of rifling and moments of inertia of the shell. In all cases the slope of the ϕ-curve over the first half-period, or rather more, is uniform and well determined. Since β appears to be really very small initially one may expect from theory the slope of this part of the curve to be $\frac{1}{2}\Omega$ whatever the value of κ. The agreement between this observed slope and the calculated value of Ω is satisfactory.

§ 8. *The Method of Analysis.*

The method of analysis of the $\sin\frac{1}{2}\delta$-curves will now be explained with reference to fig. 3. After the observed values of $\sin\frac{1}{2}\delta$ have been plotted against Ωt, the values

* *Loc. cit.*, p. 346, equation 4.06.

of Ωt corresponding to the ends of half-periods can be determined from the symmetry of the curve, with the exception of the start of the first half-period. This must, of course, be near the muzzle, but need not be actually at the muzzle, and its precise

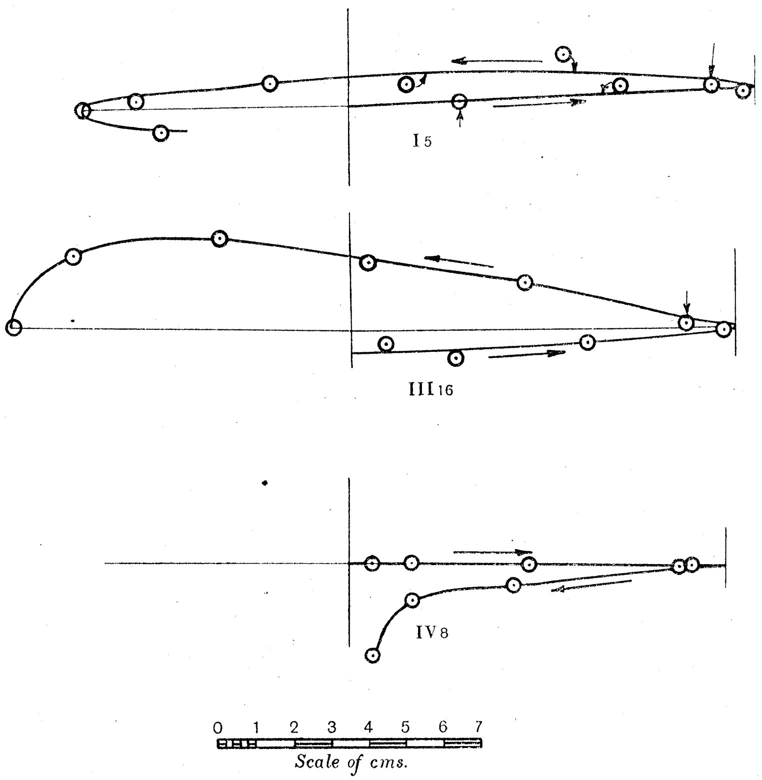

Fig. 4. Curves showing the observed angular motion of the axis with $(\sin\frac{1}{2}\delta, \psi)$ as polar co-ordinates $(\psi = \phi - \phi_0 - \frac{1}{2}\Omega t)$. The circles show the observed points and the curves are drawn freehand through them.

For I. (5) 1 cm. represents 0·01 in $\sin\frac{1}{2}\delta$.

For III. (16) and IV. (8) 1 cm. represents 0·02 in $\sin\frac{1}{2}\delta$.

position must be guessed. The values of $\sin\frac{1}{2}\alpha$ can be obtained from a rough curve drawn freehand through the observations. Each half-period is then analysed separately and the only additional constant (when $\beta = 0$) required for computing a curve, by the approximate formula

$$\sin\tfrac{1}{2}\delta = \sin\tfrac{1}{2}\alpha \,\mathrm{cn}\,(\mathrm{K} - \lambda\Omega t,\ k),$$

is the value of k or $\sin\kappa$. This may be approximated to with the help of the following artifice. Draw by eye a cosine curve (fig. 3, III. 16, first half-period) to have third order contact with the observed $\sin\frac{1}{2}\delta$-curve at the maximum and to cut the axis at T. Then the ratio of NT to the half-period NP must be $\pi/2\mathrm{K}$. For all curves of the form $y = \mathrm{A}\,\mathrm{cn}\,(\mathrm{K} - \beta x)$, for different values of k, have third-order contact at the maximum ordinate, and a cosine curve is the limiting case, in which $\mathrm{K} = \frac{1}{2}\pi$. A first estimate of κ can usually be made by this method to the nearest 5 degrees, when

$\kappa > 45$ degrees. If necessary, similar curves are drawn for other values of κ (rounds III. 11, III. 16) and the true values of κ, $\sin \frac{1}{2}\alpha$ and ΩT (the length of the half-period) finally settled by interpolation.

If required a curve can be calculated by the exact formula (10) after g has been determined by (19), but the change of shape is negligible unless $\sin \frac{1}{2}\alpha$ and, therefore, g is very large. An example of the effect of including g is shown in fig. 3 for III. (11), but round III. (11) and its companions are the only ones in which g had any sensible effect. Rounds IV. (8 and 9) illustrate the effect of a considerable alteration in b (representing the initial disturbance) in causing, when $s < 1$, a considerable change in period but only a small change in amplitude. The fit obtained between calculated and observed curves is generally good. The selected curves in fig. 3 are a fair sample of the whole.

When the minimum yaw β is not zero the curve is first calculated as above, as if $\beta = 0$, and then corrected by (33), using the observed value of β, which is obtained like α from a rough curve. An example of such curves will be found in fig. 3 for the second half-period of IV. (8) and the second and third half-periods of III. (16). After the β correction has been put on, the value of κ may require readjustment to obtain the proper fit.

The values of κ, $\sin \frac{1}{2}\alpha$ and ΩT so determined are given for each half-period of each round in Table II., together with $\sin \frac{1}{2}\beta$ and λ obtained from the equation $\lambda = K/\Omega T$. We then obtain s, q and b from equations (15)–(17).* From the values of s and q and the other observational data we can calculate $f_M \sin \delta$ as a function of v/a and δ from formulæ (3) and (4). The results are shown in figs. 1 and 2, and Tables I. and III., and have already been discussed.

The damping effects appear in the variation of the various constants from one half-period to another. In general, s increases approximately at the rate required by theory, *i.e.*, inversely as the square of the velocity at the middle point of the half-period.†

* When κ is much less than 45 degrees the method breaks down, as κ cannot be determined satisfactorily from the observational curves. The method explained in (A), p. 343, could then be employed. This is equivalent to assuming $q = 0$ and using formula (16) to determine κ given $\sin \frac{1}{2}\alpha$ and λ. Under these conditions $\sin \frac{1}{2}\alpha$ is so small that the value of q does not appreciably affect either the value of s or the shape of the curve of f_M against δ.

† Theoretically, s should increase while q should remain roughly constant. But $\sin \frac{1}{2}\alpha$ is common to the first and second half-period, while λ is determined mainly by the shape of the curve near the maximum. Hence, in general, κ alone varies between the first and second half-periods. It appears that the result of varying κ only in formulæ (15, 16) is to produce a fictitious decrease in q while the increase of s is diminished as may be seen in Table II. For this reason mean values for the whole period are used in constructing figs. 1 and 2.

TABLE II.—General Table of Results.

Column 1. Number of round and number of half-period for the round (in brackets).
,, 2. Muzzle velocity, f.s., for round or mean for group.
,, 3. Air density ρ, lb./(ft.)^3, and temperature,° F.
,, 4. Ω (= AN/B), radians/sec.
,, 5. ΩT, radians, where T is the observed duration (sec.) of each half-period.
,, 6. Observed values of $\sin \frac{1}{2}\alpha$, where α is the maximum yaw.
,, 7. Observed values of $\sin \frac{1}{2}\beta$, where β is the minimum yaw.
,, 8. Values of κ, degrees, where k (= $\sin \kappa$) is the modulus of the elliptic function which fits the observations.
,, 9. Values of λ (= K/ΩT), where K is the complete elliptic integral of the first kind for the value of κ in (8).

1	2	3	4	5	6	7	8	9
I. 8 (1)	1049	0·0792	84·2	17·5	0·276	—	79	0·172
I. 9 (1)	1084	43°	86·9	19·8	0·251	—	83	0·154
(2)				16·2	0·251	—	70	0·155
I. 10 (1)	1084		86·9	15·2	0·245	—	75	0·182
I. 17 (1)	1312	0·0812	105·2	16·6	0·192	—	75	0·167
(2)		40°		13·8	0·192	0·020	65	0·167
I. 18 (1)				13·5	0·203	—	75	0·205
(2)				12·0	0·203	0·049	70	0·209
I. 5 (1)	1582	0·0782	125·4	13·8	0·108	—	40	0·130
(2)		45°		15·6	0·108	—	57	0·133
(3)				12·0	0·070	—	20?	0·135
I. 6 (1)				13·6	0·118	—	45	0·136
(2)				14·4	0·118	—	35	0·120
I. 7 (1)				24·0	0·090	—	56	0·086
(2)				19·5	0·090	0·018	0?	0·082
II. 11 (1)	1107	0·0807	81·9	18·2	0·161	—	57	0·114
(2)		42°		14·6	0·161	0·055	20?	0·112
II. 12 (1)				20·8	0·148	—	52	0·095
(2)				16·4	0·148	0·075	35	0·105
II. 13 (1)				22·0	0·183	—	70	0·114
(2)				14·4	0·183	0·046	53	0·138

TABLE II. (continued).

1	2	3	4	5	6	7	8	9
II. 14 (1)	1334	0·0812	98·6	18·4	0·135	—	60	0·119
(2)		40°		19·8	0·135	0·014	55	0·105
II. 15 } (1)				16·6	0·122	—	48	0·117
* 16 } (2)				15·9	0·122	0·012	35	0·111
III. 11 (1)	1077	0·0807	99·0	34·0	0·332	—	84	0·107
III. 12 (1)		42°		32·0	0·290	—	85	0·120
III. 13 (1)				28·4	0·301	—	85	0·135
III. 14 (1)	1312	0·0812	120·4	22·0	0·204	—	78	0·135
(2)		40°		17·2	0·204	—	70	0·146
III. 15 (1)				18·0	0·207	—	75	0·154
(2)				14·5	0·207	0·053	55	0·141
(3)				13·9	0·187	0·053	55	0·147
III. 16 (1)				21·8	0·204	—	80	0·145
(2)				15·6	0·204	0·037	60	0·138
(3)				17·0	0·179	0·037	40	0·105
IV. 10 (1)	884	0·0807	62·5	16·5	0·233	—	70	0·152
(2)		42°		13·3	0·233	—	50	0·145
IV. 11 (1)				21·0	0·225	—	78	0·142
(2)				15·0	0·225	—	50	0·129
IV. 12 (1)				20·0	0·207	—	77	0·145
(2)				14·5	0·207	—	62	0·153
IV. 7 (1)	1553	0·0779	109·7	15·5	0·213	—	83	0·226
IV. 8 (1)		47°		18·0	0·199	—	85	0·213
(2)				13·0	0·199	0·034	75	0·213
IV. 9 (1)				11·5	0·218	—	70	0·218
IV. 1 (1)	2130	0·0782	150·6	12·5	0·160	—	66	0·187
(2)		45°		12·9	0·160	0·018	65	0·179
IV. 2 (1)				18·0	0·157	—	82	0·187
(2)				14·2	0·157	0·013	70	0·176
IV. 3 (1)				17·0	0·160	—	80	0·185
(2)				14·8	0·160	—	75	0·187
IV. 5 (1)				15·5	0·160	—	74	0·175
(2)				13·9	0·160	0·015	70	0·180

* II. (15 and 16) are practically indistinguishable. These values refer to their mean.

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Table III.—Mean values, for each group, of s, q and $\bar{v}$; the corresponding values of $\bar{v}/a$ and $f_M(\bar{v}/a, 0)$; and the percentage spread in s. Each half-period is treated separately in each group.

Group.		s.	Percentage spread in s.	q.	$\bar{v}$.	$\bar{v}/a$.	f_M.
I. 8–10	(1)	0·896	0·8	0·213	1063	0·965	11·80
I. 9	(2)	0·932	—	0·084	1053	0·956	11·56
I. 17, 18	(1)	0·892	4·3	0·578	1297	1·181	11·61
I. 17, 18	(2)	0·908	5·5	0·597	1264	1·150	12·01
I. 5–7	(1)	1·000	2·3	0·398	1560	1·413	10·56
I. 5–7	(2)	1·007	5·6	0·507	1517	1·375	11·08
I. 5	(3)	1·060	—	—	1494	1·354	10·86
II. 11–13	(1)	0·977	3·1	0·068	1094	0·994	10·34
II. 11–13	(2)	1·014	5·5	−0·04	1075	0·977	10·30
II. 14–16	(1)	0·987	2·1	0·284	1315	1·198	10·20
II. 14–16	(2)	1·006	3·2	0·066	1286	1·171	10·47
III. 11–13	(1)	0·946	2·6	−0·119	1063	0·966	13·08
III. 14–16	(1)	0·931	1·8	0·203	1296	1·180	13·10
III. 14–16	(2)	0·959	3·6	0·118	1263	1·150	13·43
III. 15, 16	(3)	0·990	3·7	—	1239	1·128	13·52
IV. 10–12	(1)	0·934	0·3	0·146	877	0·797	10·00
IV. 10–12	(2)	0·975	3·8	0·033	866	0·787	9·79
IV. 7–9	(1)	0·852	4·5	0·793	1532	1·385	11·40
IV. 8	(2)	0·864	—	0·820	1488	1·345	11·93
IV. 1–3, 5	(1)	0·897	3·5	0·987	2102	1·905	10·80
IV. 1–3, 5	(2)	0·907	3·5	0·885	2052	1·860	11·20

The values of f_M here given are not corrected for the effect of the cards.

§ 9. *The Values of b.*

The value of b represents the initial angular velocity of the axis of the shell, for at the beginning of the first half-period rosette motion may be assumed and $\delta' = b\Omega$. The values of b for each round are given in Table IV.

Table IV.—Values of b ($= \delta'_0/\Omega$), where δ'_0 is the initial angular velocity of the axis of the shell.

Group.	Values of b.			Mean.	Group.	Values of b.			Mean.
I. 8–10	0·018	0·011	0·023	0·017	III. 11–13	0·007	0·006	0·007	0·007
I. 17, 18	0·017	0·022		0·019	III. 14–16	0·011	0·016	0·011	0·013
I. 5–7	0·021	0·023	0·009	0·018	IV. 10–12	0·024	0·013	0·013	0·017
II. 11–13	0·020	0·017	0·014	0·017	IV. 7–9	0·012	0·007	0·032	0·017
II. 14–16	0·016	0·018	0·018	0·017	IV. 1–3, 5	0·024	0·008	0·010	
						0·015			0·014

They represent the size of the initial disturbance which upsets the nose-on motion, and vary irregularly in any one group, as might be expected. Their general size, however, is remarkably consistent from group to group. The corresponding values for the stable rounds and rough values for some of the rounds here analysed have been discussed by us in a previous paper.* The further results here given confirm the statements of that paper; the mean value of b for any group is practically constant somewhere between 0·01 and 0·02 for all groups fired.

§ 10. *Allowance for the Effect of Cards.*

In obtaining figs. 1 and 2 and Table I., but not in other cases, a small correction has been made to the value of the couple to allow for the impulsive action on the shell when it strikes a card. The amount of this correction was calculated from a few special rounds fired with cards on the far screens only. The following argument indicates that the effect should be roughly proportional to the couple due to the air and independent of other factors except the number of cards and the muzzle velocity. The effect of each card may be considered as an impulsive force whose moment about the centre of gravity is roughly proportional to sin δ. If the cards are (as they were) uniformly distributed in space and therefore in time, there will be a total impulse per second which is proportional to sin δ, and if the cards are not too far apart this is equivalent to a steady couple roughly proportional to the couple due to the air. The correction worked out in the case of the stable rounds at from 3 per cent. to 4 per cent. depending on the muzzle velocity. In view of the preceding argument the same corrections are applied here.

§ 11. *Concluding Remarks.*

In view of possible future experiments it is of interest to compare the merits of the stable and unstable rounds for the purpose of this analysis. A large proportion of the rounds analysed in (A) had a stability coefficient s of about 1·8. The advantages of this are that the theory of the motion is practically complete, and in addition to the values of f_M as a function of v/a rough values of the damping factors were obtained, which could be greatly improved if a longer range were available. The maximum yaw, however, is small, so that small errors in the determination of the yaw are important, while the periods are short so that, unless the cards are close together, it is difficult to draw curves through the observation points with sufficient certainty to determine the periods accurately.

For the unstable rounds here analysed the theory is imperfect as regards the determination of damping. But a small change in the value of the couple M will

* 'Proc. Camb. Phil. Soc.,' vol. XX, p. 311, 1921.

make a large change in the type of motion, so that the method is now a very sensitive one for determining the couple, and no great accuracy of observation is required. The yaw developed is also much larger, so that values of the couple are determined over a larger range of yaws.

From the point of view of general aerodynamical theory the results form a preliminary contribution to the problem of determining the force system impressed by the air on a body moving unsymmetrically through a fluid at velocities at which the compressibility of the fluid produces marked effects.

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